“Out of nothing, I have invented a strange new universe.”

YÁNOS BOLYAI (1802 – 1860)

“All of my efforts to discover a contradiction, an inconsistency, in this non-Euclidean geometry have been without success ....”

CARL FRIEDRICH GAUSS (1777 – 1855)

Instructor: Dr. Myrtle Lewin
Buttrick 325, X 6201  mlewin@agnesscott.edu
MWF 11:30 – 12:35 p.m. in Buttrick 205
Scheduled office hours (in my office): Monday and Friday 10:00 to 11:15 a.m.
Wednesday 4:00 to 6:00 p.m (except October 26) in the MLC.

• Catalog Description:
A study of axiomatic systems in geometry, including affine, projective, Euclidean and non-Euclidean geometries and the historical background of their development.

But this is very formal – so what actually will be studying?

• Some Important Themes:
A central theme of this course is to explore a mystery – why was it that Euclid’s Fifth Postulate evaded proof for more than two thousand years? The solution to this mystery is the discovery of a hidden treasure – non-Euclidean geometry. How this transformed our understanding of Euclid’s monumental work of some 2,500 years ago, is a suspense story. Our goal will be to trace this story as you retrace your experience in high school geometry, and grow to understand and appreciate it more deeply. As we go, we’ll learn a great deal about Euclidean geometry, the geometry in which we think we live (we may be in for some surprises). But we will also meet other geometries, and in particular, study spherical, hyperbolic and projective geometry. We’ll give new meaning to the word “geometry”, and develop a heightened skepticism for absolute truth.

An important focus of the course will be on discovery. Intuition (grounded in your own past experiences) will be your greatest guide. Do not suppress it. Let the text, materials on Moodle, Geometer’s SketchPad (GSP) and other technology, guide you as you allow your intuition to flourish. Problem solving will be central, as will be collaborative work and sharing ideas among colleagues. We hope you will enhance your ability to write, talk with, and listen to others about what you are thinking, and learn from each other's approaches. Don’t read a proof (in a text or from a friend) until you have given it some effort, and then try to accept only nudges. Ignore hints whenever possible, and restrain yourself from going to the “answers” provided by any text.

Another important focus of the course will be on abstraction, and the need in mathematics to work with axiomatic systems. You will develop a capacity to appreciate the role of axiomatic systems, and to become comfortable in abstract settings. We will call frequently on concepts from other courses you may have completed, and learn to interpret ideas in varieties of contexts.
Of course, a focus must be to improve your mathematical problem solving and writing skills, to develop your confidence to present mathematics to an audience, to use software as a tool in geometric thinking, as you connect your learning with your high school geometry experiences.

- **Course Objectives:** By the end of the course, each student will:
  1. understand the core concepts of both synthetic and analytic Euclidean geometry, in both 2- and 3-dimensional space;
  2. have developed an understanding of the role of investigative experiences in geometry which lead to the discovery of key geometrical relationships;
  3. have developed her skills in proof writing, and an enhanced appreciation of the power of proofs in verifying conjectures and working within an axiomatic system;
  4. have gained a historical perspective of the development of geometric ideas in a variety of cultures, including the development of non-Euclidean geometry;
  5. be comfortable using technology to aid discovery in geometry, including the Geometer’s SketchPad software.

- **Required Text and materials:**

  ⇒ The required text is *Elementary Geometry from an Advanced Standpoint* by Edwin E. Moise, Third Edition, Addison Wesley, 1990. This book is not out of date mathematically, even though the style of writing is a little old fashioned. But it captures the materials we need to study well, carries the reader into abstraction from an intuitive (let’s capture what we hope to find) vantage point, the writing is careful and clear, and the problem sets are generally well done (I’ll add problems to sections regularly). With adequate supplementary materials to bring technology into your geometry learning, I’m hoping that you’ll enjoy using this text.

  ⇒ You should have access to *The Geometer’s Sketchpad (GSP), Version 5*¹, a software package from Key Curriculum Press. It is available on selected campus computers (we have a site license for 10 copies). For the student version² see [http://www.keypress.com/x26810.xml](http://www.keypress.com/x26810.xml) where there are lots of additional resources too. GSP is used extensively in high schools too, so may be a good investment if you are planning to teach. Unfortunately, the college’s site license does not permit any copies to be placed on your own computer.

  ⇒ Almost all course materials are available on the Math 314 class page on Moodle (including this document where you may click on the sites listed above). You are expected to consult this Moodle site regularly. Resources there include announcements, links to resources, assignments and handouts. Email messages are also an official means of communication.

  ⇒ You will need a supply of fine pencils, straight edges/rulers (preferably a short, transparent one and one longer than 12 inches), a pair of compasses (sometimes called a compass), and colored pens (these are essential, with fine points, lots of easy-to-identify colors) and unlined paper (use the scratch paper supplies around campus).

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¹ There is one copy in Buttrick 205 (our classroom), one on the math lobby computer, one in Buttrick G-13 (across the hallway from the MLC) and one on each of the six computers in the MLC. There is one remaining copy that we can have uploaded where you decide is a good location.

Some recommended texts
You are encouraged to get into the habit of browsing through other texts, and readings may be given from other sources from time to time. Texts we will be using will include those listed below, and others (you may find your own). Some are available in the McCain Library, some in my lending library (on my desk or in the MLC, Buttrick G-12). Feel free to borrow them from me. But please do not remove any of my books that are in the MLC from that room without my permission.

⇒ *A Course in Modern Geometries* by Judith N. Cederberg (Springer-Verlag Undergraduate Texts on mathematics series). This is an excellent book, its approach is algebraic. It provides an alternate development of projective geometry which you might find useful to study;

⇒ *College Geometry* by Howard Eves (Jones and Bartlett, Publishers). This text is a masterpiece, but it is not easy. For anyone who plans to teach high school mathematics, it is an excellent resource with interesting historical insights. All of Eves' books are wonderful;

⇒ *Excursions into Mathematics* by Beck, Bleicher and Crowe (Worth, 1969, reprinted in paperback by A.K. Peters, 2000) (in McCain). Chapter 4, entitled “Some Exotic Geometries”, provides an informal discussion of some of the material we'll cover in chapter 24 of Moise;

⇒ *Euclidean and Non-Euclidean Geometries, Development and History*, Third Edition, by Marvin Jay Greenberg, W.H. Freeman & Co., 1993. This was the prescribed text for MAT 314 in 2005. It is the best organized, most thorough and reliable geometry text at the undergraduate level that I have ever seen. It is clearly written, with lots of historical material interwoven into the development. But it is not easy, and the organization of the book, with problems at the end of chapters, makes it very hard for the student. So I have reluctantly abandoned it. A copy will be in the Math Learning Center (Buttrick G-12). For students planning to go to graduate school in Mathematics, it is an excellent resource.

⇒ *College Geometry, A Discovery Approach with the Geometer's Sketchpad*, by David C. Kay, Addison Wesley, 2001. This book has some excellent exercises using GSP, and it is clearly written. But it doesn’t take the axiomatic approach that we need, so its use is limited.

⇒ *Euclidean and Transformational Geometry, A Deductive Inquiry*, Shlomo Libeskind, Sudbury MA, Jones and Bartlett Inc., 2008. This book has some excellent exercises using GSP, and it is also clearly written.

These books are on a lighter note, recommended for browsing rather than study:

⇒ *Journey into Geometries* by Marta Sved, with a foreword by H.S.M.Coxeter, MAA (the Mathematical Association of America) 1991. This text introduces hyperbolic geometry through the fictitious characters of Lewis Carroll's Alice, and a character called Dr. Whatif


⇒ *The Non-Euclidean Revolution*, by Richard J. Trudeau (Birkhauser Boston, 1987). This text won an award for expository writing. It is in McCain, I’ll refer to it occasionally;

⇒ *To Infinity and Beyond: A Cultural History of the Infinite* by Eli Maor, Birkhauser Boston, 1987, reprinted in paperback by Princeton University Press, 1991. This is a beautiful book (it's in McCain, and my copy is in the MLC). Among other things, Chapter 16, “The Vain Search for Absolute Truth”, is important reading.
• Office Hours etc:

The course will be highly interactive, and sharing ideas in and out of class is a useful way to learn how to think creatively and communicate coherently. GSP is a wonderful tool to help you make conjectures and develop insight – learn to use it. And think with a friend! When you turn in work for a grade, please acknowledge any collaboration (including hints and proofs and answers in the text), but you are expected to write your own solutions to everything (Honor Code). Journal notes to yourself as you work through problems are helpful - something like “Stuck here”, “This seems to use the same technique as in …”, must think about this again…”, “Jane had an idea to do …”, etc. are useful.

My scheduled office hours (in my office) are Monday and Friday from 10:00 to 11:15 a.m. and Wednesday from 4:00 to 6:00 p.m (except on October 26) in the MLC. But you are invited to call me, or pop in to schedule meetings with me in my office at your convenience, either alone or with a friend. Because I am not working full time, I'm trying to keep Tuesdays and Thursdays open for other things, so prefer MWF meeting times. And sometimes a telephone call to me in my office or email conversation will provide you with the nudge you need.

Sometimes I will spend time in the MLC in room G–12 Buttrick. Also, the MLC is a wonderful place for you to come and work alone or with others (including me), and chat about your progress. Students in this course who are also LA’s will do this anyway, so why not join in? Research shows convincingly that students do better in math courses when they develop collaborative relationships and use them wisely.

If you have a disability that may have some impact on your work in this class and for which you may require accommodations, please see Machamma Quinichett in the Office of Academic Advising to register for services. If you have received an accommodation checklist, please meet with me as soon as possible to discuss the provisions of those accommodations.

• Some comments on the Moise book:

The heart of the course for both MAT 314 will be in chapters 2 through 12, 15, and 24. Chapter 1 is mostly a review of topics in MAT 204, or whatever course you took that introduced you to proof writing and abstraction. As we work through the early chapters, you will develop your ability to explore ideas and make conjectures, largely using GSP.

The course begins with the development of an axiomatic system for what we might naively believe is Euclidean geometry – but as we will soon learn, life is more complicated than this. In Chapters 2 through 8, we will develop what is known as “absolute geometry” or “neutral geometry”. Then in chapters 9 through 12, we get a feel for both Euclidean geometry and what we mean by non-Euclidean geometries, including Spherical and Hyperbolic geometry.

Having now understood that there is no such thing as a universal “Geometry”, we study two topics:
(i) Projective geometry (from notes that will be provided)
(ii) Hyperbolic geometry (largely in chapter 24 of Moise)

In any time that may be left, we’ll study Chapter 15, some three dimensional Euclidean geometry, and Chapter 18, an introduction to isometries.
• **Tests, Assignments and Grading:**

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<tr>
<th>Assignment</th>
<th>Points</th>
<th>Percentage</th>
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<tbody>
<tr>
<td>Class involvement</td>
<td>30</td>
<td>6%</td>
</tr>
<tr>
<td>Regular reading “pop” quizzes</td>
<td>40</td>
<td>8%</td>
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<tr>
<td>Weekly problem sets, usually due Wednesdays at 6:00 p.m.</td>
<td>160 (best 8)</td>
<td>32%</td>
</tr>
<tr>
<td>GSP project</td>
<td>30</td>
<td>6%</td>
</tr>
<tr>
<td>Midterm test (open book problem set)</td>
<td>120</td>
<td>24%</td>
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<tr>
<td>Final exam (open book problem set)</td>
<td>120</td>
<td>24%</td>
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<tr>
<td><strong>Total</strong></td>
<td><strong>500</strong></td>
<td><strong>100%</strong></td>
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I will use a ten point scale for this course, using + and - as appropriate. But in order to get a final grade of A- (resp. B-, C-, pass) or better, you need an average of at least 84% (resp. 73%, 62%, 50%) on the two problem sets. Assuming you meet this requirement, 90% guarantees you an A-, 80% a B-, 70% a C-, and 60% a passing grade.

• **Class Involvement (30 points):**

Your ability and willingness to participate intelligently in class is the first measure of your learning and understanding. You are part of a learning community, and we all share the responsibility for the welfare of that community. This grade will be given in large part on the regularity of your attendance, the quality of your suggestions and solutions to problems, your capacity to listen respectfully to what others have to say and to involve others in your classroom conversations, and your insights during class. You are expected to be prepared, to do the reading as expected, and to have tried many of the problems that are assigned in time to contribute to class discussion. Tardies and unexcused absences will count against you here. And it is always better to come late to class than to skip a class. Please send me an email if you have to miss a class – it is a courtesy, given the size of the class. We really will notice if you are not present.

• **Regular Reading Quizzes (40 points):**

From time to time, I’ll ask you to respond to some items related to the reading that was expected for that class (a pop quiz, at the beginning of class, much more informal than the quizzes in MAT 204). You may be asked for a definition, the statement of a theorem, or something else that you should have learned or understood. There will be at least ten of these. Each will be graded S+, S, S – or U. Your best eight grades will count. For 40 points, you need eight S+’s. Eight S’s will give you 32 points, and eight S – ‘s will give you 24 points.

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3 More specifically, I use the standard college grading scale, as follows:

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<thead>
<tr>
<th>Grade</th>
<th>Percentage</th>
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<tbody>
<tr>
<td>90 to 100 A</td>
<td>90 to less than 93 A-</td>
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<tr>
<td>87 to less than 90 B+</td>
<td>83 to less than 87 B</td>
</tr>
<tr>
<td>80 to less than 83 B-</td>
<td>77 to less than 80 C+</td>
</tr>
<tr>
<td>73 to less than 77 C</td>
<td>70 to less than 73 C-</td>
</tr>
<tr>
<td>67 to less than 70 D+</td>
<td>63 to less than 67 D</td>
</tr>
<tr>
<td>60 to less than 63 D-</td>
<td>Less than 60 F</td>
</tr>
</tbody>
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Weekly Problem Sets (160 points):

One of the important roles of graded work is the conversation that it enables between you (the novice learner) and me (the expert learner). Use these problem sets as a learning tool, allowing yourself to acknowledge when the going gets rough, and allowing others in the class (including your instructor) to intervene, share ideas, give you that nudge that lets you persevere again and experience those Aha! moments.

The Weekly Problem Sets will hopefully capture a representative selection of the problem solving you are doing. They consist of specific problems (often drawn from class discussion), given to you most weeks by Friday’s class (either in class or posted on Moodle earlier), due the following Wednesday by 6:00 p.m. This work will be checked and returned to you, by Friday in class if possible. Each problem set will be graded A/B/C/U (with +/-), and will not be accepted late. I encourage you to turn in responses to my comments (corrections, if you prefer that language), due within one week of my returning the graded first attempt. The original must accompany any resubmitted work. If these corrections are well done, your grade on the problem set can increase by up to one letter (from a B- to an A-, for example). You will be given a problem set envelope (you are used to this routine from MAT 204). Work will be accepted only if it is turned in to me in your envelope, properly organized.

Your best 8 problem sets will count, although you will hopefully choose to turn them all in (You can expect at least nine of them to be given). 8 A's will give you 152 points, 8 B's will give you 128, 8 C's 104, and a U will not count. (So 8 A+'s give you 160 points).

Occasionally, there will be a challenge problem which you are encouraged to solve. Please work on these problems on your own, and let me see what you do. You may earn up to 10 bonus points for this work (this is merely an incentive – I hope you'll try them anyway!) – but the max is 160 points. These 160 points represent about one third of the course grade, and provides an opportunity for thoughtful, regular work with detailed feedback. You might also note that 128 out of 160 is only 80%.

GSP project (30 points) – deadline December 02:

Submit a Geometer’s Sketchpad file (hopefully with multiple pages) that show some kind of creative way in which you have learned to use GSP as a tool in your exploration of geometry. There are lots of ways you might do this – for example, you might exhibit the steps in the solution of a fairly complex problem (such as solving the Nine Point problem), or you may use GSP to produce a picture of an architectural structure (like the Brooklyn Bridge).

Timeline:
⇒ Rough draft submitted by November 11 (in enough time for you to change topic if necessary);
⇒ Brief (about 5 minutes) presentation in class on November 30 or December 02;
⇒ GSP file submitted by 5:00 p.m. December 02.

The Midterm Test and Final Exam: (collectively 48% of your final grade)

The purpose of these problem sets is to make sure that you are developing a broad understanding of the ideas of the course, and have put them together coherently in your mind. While you may study and solve problems collaboratively most of the time, bouncing ideas off both other students and your instructor, there is a need to ensure that you are learning to think independently, developing a body of

\[4\] Due dates are on Wednesdays 9/07, 9/14, 9/21, 9/28 and 10/05 before the midterm exam and Fall Break; on 10/02, 11/09, 11/16, after Fall Break but before Thanksgiving, and 11/30 after Thanksgiving.
knowledge about geometry, and can problem solve alone with a time constraint. I also want to encourage you to take pretty much all the homework I suggest seriously, not only what you turn in, because some problems on these Problem Sets will be drawn from this homework. You will have had lots of time to ask questions, work with study buddies, etc., on problems before they and others appear on these two solo problem sets. You should treat these like take-home tests, working entirely on your own, except that you have access to your notes and text.

**Midterm Test (120 points):**

This will be given to you before fall Break (on Monday October 10 in class), and will be due on **Friday October 22 by 11:30 a.m.** (at the beginning of class). You may complete this problem set in several different sessions over several days, there is no time constraint (recommended time about four to six hours), but you are expected not to speak to anyone about these problems, nor to consult any texts other than the Moise text and your notes, from the time you open your envelope until you are notified that all test envelopes have been turned in to me.

**Final Exam (120 points):**

The Final Problem Set has essentially the same constraints as the midterm problem set. It will be given to you in the last class meeting (Monday December 5), and due at the end of the exam period (Tuesday December 13 at 5:00 p.m.).

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**Academic Integrity and Collaboration:**

Integrity is at the heart of successful mathematics study, just as collaboration, the sharing of ideas and inspirations, is at the heart of how mathematicians work effectively. We are constantly asking ourselves whether we really understand, really control the techniques, and have really expressed our thoughts in our own voices. Of course, you are expected to pledge all your work turned in to me. But even in your own work, especially when you have shared ideas with others, you need to make sure that you are in control, that your work reflects your own understanding of the material, and that you are critical enough to know when you are stuck. Take advantage intelligently of the ways in which germs of ideas can develop by virtue of collaboration, but acknowledge the ideas of others (as you would a text or web page in a writing-intensive course). And when you pledge your work, you are telling me that your writing reflects your own understanding.

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**Student course evaluations:**

Your feedback on the course is extremely valuable to me, the mathematics department, and the administration. I take student comments very seriously and use them to improve the course the next time I teach it, so you are beneficiaries of students’ insights in previous years. You are responsible for completing an evaluation of the course at the end of the semester. The evaluation takes place on-line, and you complete it outside of class. More details will be provided.