It seems to me that the poet has only to perceive that which others do not perceive, to look deeper than others look. And the mathematician must do the same thing.

SONYA KOVALEVSKAYA

How can it be that mathematics, being after all a product of human thought independent of experience, is so admirably adapted to the objects of reality?

ALBERT EINSTEIN

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Class times: Monday Wednesday 4:00 – 5:15 p.m.
Friday 2:00 – 2:50 p.m.

Room: Buttrick G – 13

• Course Objectives and Course Structure:

A central theme of this course is to explore a mystery – why was it that Euclid’s Fifth Postulate evaded proof for more than two thousand years? The solution to this mystery is the discovery of a hidden treasure – non-Euclidean geometry. How this transformed our understanding of Euclid’s monumental work of some 2,500 years ago, is a suspense story. Our goal will be to trace this story as you retrace your experience in high school geometry and trig, and grow to understand and appreciate it more deeply. As we go, we’ll learn a great deal about Euclidean geometry, the geometry in which we think we live (we may be in for some surprises). But we will also meet other geometries, and in particular, study spherical, hyperbolic and projective geometry. We’ll give new meaning to the word “geometry”, and develop a heightened skepticism for absolute truth.

The first important focus of the course will be on discovery. Intuition (grounded in your own past experiences) will be your greatest guide. Do not suppress it. Let the texts, materials on Blackboard, and me guide you as you allow your intuition to flourish. Problem solving will be central, as will be working collaboratively and sharing ideas among yourselves. We hope you will enhance your ability to write and talk about what you are thinking (often by drawing or modeling it) and learn from each other’s approaches. Don’t read a proof (in a text or from a friend) until you have given it some effort, and then try to accept only nudges. Ignore hints whenever possible, and restrain yourself from going to the “answers” provided by any text. (This is one of the lessons that Carol Schumacher taught you.)

The second important focus of the course will be on abstraction, and the need in mathematics to work with axiomatic systems. You will develop a capacity to appreciate the role of axiomatic systems, and to become comfortable in abstract settings. This training should prepare you for other upper division courses (and graduate school), as it feeds off courses you may have done or are doing. It should also help you to see all your mathematical studies as part of a whole. We will call frequently on concepts from other courses you may have completed (the art of mathematical thinking, linear algebra and multivariable calculus in particular, and occasionally abstract algebra, although not all of these are prerequisites), and learn to interpret ideas in varieties of settings.

Central objectives are also to improve your mathematical problem solving and writing skills, to develop your confidence to present mathematics to an audience, to use software as a tool in geometric thinking, and to connect your learning with your high school geometry experiences. You will be asked, frequently, how what you have learned or done can guide you as a teacher of novice learners (like high school students). In this way, if you allow yourself to think in this way, you will be preparing to be an excellent, reflective math teacher.
• **Required Text and materials:**

⇒ The required text is *Elementary Geometry from an Advanced Standpoint* by Edwin E. Moise, Third Edition, Addison Wesley, 1990. This book is not out of date mathematically, even though the style of writing is a little old fashioned. But it captures the materials we need to study well, carries the reader into abstraction from an intuitive (lets capture what we hope to find) vantage point, the writing is careful and clear, and the problem sets are generally well done (I'll add problems to sections regularly). With adequate supplementary materials to bring technology into your geometry learning, I'm hoping that you'll enjoy using this text.

⇒ You should have access to *The Geometer's Sketchpad (GSP), Version 4.06*, a software package from Key Curriculum Press. It is available on selected campus computers in G-12 Buttrick. We have a site license for 10 copies. A tutorial written by David Royster, UNC Charlotte, is available at [http://education.uncc.edu/droyster/PMET/droyster/GSP%20Tutorial.pdf](http://education.uncc.edu/droyster/PMET/droyster/GSP%20Tutorial.pdf). The student version sells for $39.95 at [http://www.keycollege.com/catalog/titles/sketchpad](http://www.keycollege.com/catalog/titles/sketchpad), where there are lots of additional resources too. It is used extensively in high schools too, so may be a good investment if you are planning to teach.

⇒ Almost all course materials are available on the Math 314 class page on Blackboard (including this document where you may click on the sites listed above). You are expected to consult this Blackboard site regularly. Resources there include announcements, links to resources, assignments and handouts. Email messages are an official means of communication.

⇒ You will need a supply of fine pencils, straight edges/rulers (preferably both a short, transparent one and one longer than 12 inches), a pair of compasses (sometimes called a compass), and colored pens (these are essential, with fine points, lots of easy-to-identify colors). Some unlined paper (use the scratch paper supplies around campus) and scissors will be useful too.

• **Some comments on the Moise book:**

The heart of the course will be in chapters 2 through 11, and in chapters 24, 25 and 26. Chapter 1 is mostly a review of topics in MAT 204. We'll also study Projective Geometry from notes that will be provided. Greenberg’s text will be used as a reference frequently. You are encouraged to get into the habit of browsing through texts, and readings will be given from other sources from time to time. Texts we will be using will include those listed here, and others (you may find your own). Some are available in the McCain Library, some in my lending library (on my desk or in Buttrick G-12). Feel free to borrow them from me. But please do not remove any of my books that are in Buttrick G-12 from that room.

• **The following are recommended texts**

⇒ *A Course in Modern Geometries* by Judith N. Cederberg (Springer-Verlag Undergraduate Texts on mathematics series). This is an excellent book, its approach is algebraic. It provides an alternate development of projective geometry which you might find useful to study;

⇒ *College Geometry* by Howard Eves (Jones and Bartlett, Publishers). This text is a masterpiece, but it is not easy. For anyone who plans to teach high school mathematics, it is an excellent resource with interesting historical insights. All of Eves' books are wonderful;
Excursions into Mathematics by Beck, Bleicher and Crowe (Worth, 1969, reprinted in paperback by A.K. Peters, 2000) (in McCain). Chapter 4, entitled “Some Exotic Geometries”, provides an informal discussion of some of the material we'll cover;

Euclidean and Non-Euclidean Geometries, Development and History, Third Edition, by Marvin Jay Greenberg, W.H. Freeman & Co., 1993. This was the prescribed text in 2005. It is the best organized, most thorough and reliable geometry text at the undergraduate level that I have ever seen. It is clearly written, with lots of historical material interwoven into the development. But it is not easy, and the organization of the book, with problems at the end of chapters, makes it very hard for the student. So I have reluctantly abandoned it. A copy (of an old edition) will be in the Math Learning Resource Center (Buttrick G-12). Please leave it there.

College Geometry, A Discovery Approach with the Geometer's Sketchpad, by David C. Kay, Addison Wesley, 2001. This book has some excellent exercises using GSP, and it is clearly written. But it doesn’t take the axiomatic approach that we need, so its use is limited. I'll refer to it occasionally


These books are on a lighter note, recommended for browsing rather than study:

Journey into Geometries by Marta Sved, with a foreword by H.S.M.Coxeter (published by the MAA, the Mathematical Association of America, 1991). This text introduces hyperbolic geometry through the fictitious characters of Lewis Carroll's Alice, and a character called Dr. Whatif;


The Non-Euclidean Revolution, by Richard J. Trudeau (Birkhauser Boston, 1987). This text won an award for expository writing. It is in the library, I'll refer to it occasionally;


Office Hours and Sources of help:

The course will be highly interactive, and sharing ideas in and out of class is a useful way to learn how to think creatively and communicate coherently. GSP is a wonderful tool to help you make conjectures and develop insight – learn to use it. And think with a friend! When you turn in work for a grade, please acknowledge any collaboration (including hints and proofs and answers in the text), but you are expected to write your own solutions to everything (Honor Code!), keep your work properly organized, and be able to find things quickly. Journal notes to yourself as you work through problems are helpful - something like “Stuck here”, “This seems to use the same technique as in …”, must think about this again…”, “Jane had an idea to do …”, etc. are useful.

Please use my open door, self-scheduled office hours, email or call me as needed. You all know how to find me. You are also free to chat with me when I’m in the Math Learning Resource Center. I plan to spend one or more hours there regularly each week.
Test, Assignments and Grading:

Class involvement: 30 points
Regular reading quizzes: 30 points
Weekly homeworks: 150 points
Paper / Presentation: 40 points
Midterm problem set: 100 points
Final problem set: 100 points
Total: 450 points

I will use a ten point scale for this course, using + and - as appropriate. But in order to get a final grade of A- (resp. B-, C-, pass) or better, you need an average of at least 84% (resp. 74%, 64%, 54%) on the two problem sets. Assuming you meet this requirement, 90% guarantees you an A-, 80% a B-, 70% a C-, and 60% a passing grade.

Class Involvement:
Your ability and willingness to participate intelligently in class is the first measure of your learning and understanding. You are part of a learning community, and we all share the responsibility for the welfare of that community. This grade will be given in large part on the regularity of your attendance, the quality of your suggestions and solutions to problems, your capacity to listen respectfully to what others have to say and to involve others in your classroom conversations, and your insights during class. You are expected to be prepared, to do the reading as expected, and to have tried many of the problems that are assigned in time to contribute to class discussion. Tardies and unexcused absences will count against you here. (30 points)

Regular Reading Quizzes:
From time to time, I'll ask you to respond to some items related to the reading that was expected for that class (a pop quiz, much more informal than the quizzes in MAT 204). You may be asked for a definition, the statement of a theorem, or something else that you should have learned or understood. There will be at least seven of these. Each will be graded S+, S, S- or U. Your best five grades will count. For 30 points, you need five S+'s. Five S’s will give you 25 points, and five S-’s will give you 20 points. (30 points)

Weekly Homeworks:
You will be expected to turn in specific problems most weeks (often drawn from class discussion), given to you on Wednesday, due the following Monday by 6:00 p.m. This work will be checked and returned to you, by Wednesday if possible. Each homework assignment will be graded A/B/C/U (with +/-), and will not be accepted late. I encourage you to turn in responses to my comments (corrections, if you prefer that language), due within one week of my returning the graded first attempt. The original must accompany any resubmitted work. If these corrections are well done, your grade on the homework will go up one letter (from a B- to an A-, for example). You will be given a homework envelope (you are used to this routine from MAT 204). Homework will be accepted only if it is turned in to me in your homework envelope, properly organized.
Your best 8 homeworks will count, although you will hopefully choose to turn them all in (You can expect at least nine\(^1\) of them to be given). 8 A's will give you 140 points, 8 B's will give you 116, 8 C's 92, and a U will not count. An additional ten points are there for some unusual additional problem solving – in response to some challenge problems, perhaps – or several A+'s, or other exceptional work. These 150 points represent one third of the course grade, and provides an opportunity for thoughtful, regular work to serve you well. You might also note that 116 out of 150 is only 77%, so an average of a B on your best 8 homeworks is not really a B. \(\text{(150 points)}\)

- **Class paper/presentation:**

You will be expected to prepare a topic selected from a mathematics journal or text, but that goes outside of what was discussed in class, and to talk about that topic to the class (taking between 15 and 20 minutes in a class period including questions). The material you present should also be written\(^2\) in the form of a self-contained, useful handout, in the style of a section from a math text. You may want to give homework to your classmates – chat with me about that, and you may want to work with a partner in preparing a topic that can easily be broken into two presentations. Some references should be given. (Recommended length: 2 to 4 pages, including figures if appropriate.) The purpose of this is to teach you to study a section of new mathematics independent of the classroom environment, and to present mathematics to an audience in a coherent, interesting and approachable manner.

The topic selection deadline is Monday October 29 at 6 p.m. (after the first problem set has been handed in). Your topic must be approved by me by that date. So you are advised to chat with me well before then. The presentations will be completed before Thanksgiving if at all possible. You are encouraged to share your ideas with me before presenting them to the class. The schedule will be set on Friday November 2, in class.

Possible resources include the recommended texts listed above, The Mathematical Intelligencer, College Mathematics Journal, Mathematics Magazine, (all available in the library), and the WWW. See also E-Handout # 2 on Blackboard. You will be graded on the appropriateness of your topic, the clarity of your presentation, the quality of your handout, and your willingness and ability to respond to questions. \(\text{(40 points)}\)

- **The Midterm and Final Problem Sets:**

The purpose of these problem sets is to make sure that you are developing a broad understanding of the ideas of the course, and have put them together coherently in your mind. While you may study and solve problems collaboratively most of the time, bouncing ideas off both other students and your instructor, there is a need to ensure that you are learning to think independently, developing a body of knowledge about geometry, and can problem solve alone under pressure. I also want to encourage you to take pretty much all the homework I suggest seriously, not only what you turn in, because some problems on these Problem Sets will be drawn from this homework. You will have had lots of time to ask questions, work with study buddies, etc., on these problems before they appear on these two Problem Sets. You should treat these Problem Sets like take-home tests, working entirely on your own.

\(^1\) In fact, I've scheduled ten, to provide flexibility. You might elect to skip one when you are preparing your presentation, for instance.

Due dates on 9/10, 9/17, 9/24, 10/01, 10/08, 10/15, 11/05, 11/12, 11/19, and 12/03.

\(^2\) Use Microsoft word, and its equation editor for mathematical expressions.
Midterm Problem Set:
This will be given to you before fall Break (on Wednesday October 17), and will be due on Friday October 26 at the beginning of the class (4:00 p.m.). You may complete this problem set in several different sessions over several days, there is no time constraint (recommended time about four hours), but you are expected not to speak to anyone about these problems, nor to consult any texts other than the Moise text, from the time you open your envelope until all problem sets have been turned in to me. (100 points, about 22% of the final grade)

Final Problem Set:
The Final Problem Set has essentially the same constraints as the midterm problem set. It will be given to you in the last class meeting (Monday December 10), and due at the end of the exam period (Tuesday December 18 at 5:00 p.m.). (100 points, about 22% of the final grade)

• Academic Integrity and Collaboration:
Integrity is at the heart of successful mathematics study, just as collaboration, the sharing of ideas and inspirations, is at the heart of how mathematicians work effectively. We are constantly asking ourselves whether we really understand, really control the techniques, have really expressed thoughts in our own voices. Of course, you are expected to pledge all your work turned in to me. But even in your own work, especially when you have shared ideas with others, you need to make sure that you are in control, that your work reflects your own understanding of the material, and that you are critical enough to know when you are stuck. Take advantage intelligently of the ways in which germs of ideas can develop by virtue of collaboration, but acknowledge the ideas of others (as you would a text or web page in a writing-intensive course). And when you pledge your work, you are telling me that your writing reflects your own understanding.

• Some dates and deadlines:

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<thead>
<tr>
<th>Date / Time</th>
<th>Event</th>
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<tbody>
<tr>
<td>Wednesday most weeks</td>
<td>Weekly homework assigned to turn in Monday</td>
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<tr>
<td>Mondays most weeks</td>
<td>Homework due, in your envelope</td>
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<tr>
<td>Wednesday October 17, in class</td>
<td>Midterm problem set given to you</td>
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<tr>
<td>Friday October 26, 4:00 p.m.</td>
<td>Midterm problem set due</td>
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<tr>
<td>Monday October 29, 6:00 p.m.</td>
<td>Topic selection deadline for presentation</td>
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<td>Friday November 2, in class</td>
<td>Presentation schedule set</td>
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<td>Monday December 10, in class</td>
<td>Final problem set given to you</td>
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<tr>
<td>Tuesday December 18, 5:00 p.m.</td>
<td>Final problem set due</td>
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