Math 321: Abstract Algebra

Fall, 2004
Monday, Wednesday, and Friday, 8:45-9:50 a.m.
Buttrick G-12

Dr. Alan Koch
e-mail: akoch@agnesscott.edu

Office: Buttrick 329
Phone: (404) 471-6223

Office Hours. Mondays, 3:00-4:00 and Wednesdays, 11:30-12:30 or by appointment.

Course Information

Prerequisites. Math 204 (The Art Of Mathematical Thinking) and Math 206 (Linear Algebra).

Text. *A First Course in Abstract Algebra*, 7th edition, by John B. Fraleigh.

Course Content. Chapters 1-7 of the text will be studied. Topics include groups, factor groups, cyclic groups, rings, prime and maximal ideals, fields, homomorphisms and isomorphisms, factorization, and field extensions.

What is "Abstract Algebra"? Well, first we ask, "what is algebra?" Simply put, algebra is about solving equations. In high school algebra, all of the equations have real variables, i.e. variables whose values are assumed to be real numbers. This is rather restrictive, for there are many other very nice number systems out there. In abstract algebra, we try to solve equations where the variables come from *different* number systems. Much of this class consists of asking, "what other number systems are out there?"

What good is "Abstract Algebra"? At first, it may seem like this class is an exercise in abstract mathematical thinking. However, there are many applications to what we will be studying. Abstract algebra plays a vital role in:

- Automata: the development of finite-state machines, for example telephones and vending machines.
- Business and Economics: maximizing a set of linear equations constrained by a set of linear inequalities.
- Computer Graphics: 3-D rotations, movement of figures, etc.
- Cryptography: creating and breaking secret codes.
- Information Theory: the design of efficient error-correcting codes.
- Physics: the state of sub-atomic particles.
- Quantum Chemistry: the movements of electrons.

Course Goals. By the end of the semester, you should:

- Understand the notion of abstract algebra.
  There is a lot of material to learn this semester. The hope is that you will take from this class the "philosophy" of abstract algebra. There will be an occasional test question that will help measure this – for example there might be a question that introduces a new number system we hadn't studied.

- Be able to write a concise algebra proof.
  This is where you put your Math 204 training into action. The assignments will consist largely of proofs.

- Be able to read and understand technical mathematics
  The presentation, described below, will be a direct way of assessing this, but you should also be reading in the book as the class goes on.

Class Description. With the exception of the days mentioned below, this will be conducted as a lecture-style class. That is not to say your input won't be valued or solicited: frequently questions will be posed of you, and (hopefully) questions will be posed of me. If it any time you have a question, please feel free to speak up rather than raise your hand. That's a power trip I just don't need.

"Q&A". Six days are designated "Q&A", each occurring at the completion of a chapter. On these days, we will have time for you to both ask and answer questions. We will take as many questions as possible.

Attendance. You are expected to attend every class. Attendance does not have a direct impact on your grade (except during oral presentations). It will, however, have an indirect and

impact on your grade. Trust me. Also, you will not be penalized directly for tardiness, however you are expected to arrive to each class on time.

Blackboard. A Blackboard site has been set up for this course. Here you will find all the handouts for the course. Most will be .pdf files, so make sure the computer you're using has Adobe Acrobat installed.

Assessment

Here's how you'll be graded…

Homework. Each night, there will be homework problems assigned from the sections covered during the lecture. They will consist (primarily) of odd problems from the book, so you can check your answers in the back. This homework will not be collected, but it is assumed that it will be completed by the start of the next class. (Obviously, you won't be graded on this, but it seemed the most logical place to put this information.)

Assignments. There will be five assignments that you will turn in. While the questions will tend to be more theoretical than the problems assigned after each section, it is important that you do the daily homework to get a feel for what is going on before you attempt the assignments. Assignments are due by the start of class on the day indicated at the top of the problem set. Late assignments will not be accepted. Each is due *in class* at the start of class on the date indicated at the top of the assignment.

You are encouraged to discuss these assignments with the others in the class, but your write-up must be your own. If you have any question about this policy, please let me know.

Exams. You will have three take home exams. The exams will tend to focus more on the calculation-type problems than the theoretical, although this should not be taken as a guarantee. These are (tentatively) scheduled to be handed out September 22, October 27, and November 17. They will be due at the start of the next class. If you have a conflict with any of these dates, let me know ASAP. (The day after the exam is not ASAP.) The exams will cover material from the text, along with material presented in class.

Final Exam. The final exam is cumulative. Surprise, surprise.

Honor Code. All students are expected to follow the honor code throughout the semester. Any graded work, be it an assignment or an exam, must be pledged (and signed) in order for it to be graded. Please consult the student handbook for more details.

Oh, one more thing…

Oral Presentation and Paper. In addition to the assignments listed above, you (or pairs of you) must also complete an oral presentation and a paper. The purpose of this is to

(a) give you more exposure to an application of abstract algebra
(b) provide you with an easy forum to read a scientific document
(c) offer you practice in formal mathematical lecturing
(d) bestow you with the experience of writing

The presentations will take place during three of the last four days of class. It is expected that you will work in pairs (in which case both persons must talk for roughly the same amount of time), however you may work by yourself provided there are enough topics to go around.

Once a topic is picked, you are to prepare a 20-minute talk on the subject. You are to teach your topic to the rest of the class at a level consistent with the level of someone who has just successfully completed abstract algebra. It is helpful to provide basic definitions, give examples, etc. The topics are as follows:

[redacted]	**Group Actions and Counting**
Cayley Digraphs	[redacted]
[redacted]	**Series of Groups**

You will (hopefully) not have enough time in 20 minutes to completely describe your topic. Be prepared to construct a talk that lasts 20 minutes. Make sure it has a definite conclusion – do not just keep talking and talking until your time is up. Be prepared to take questions from the class (including from me).

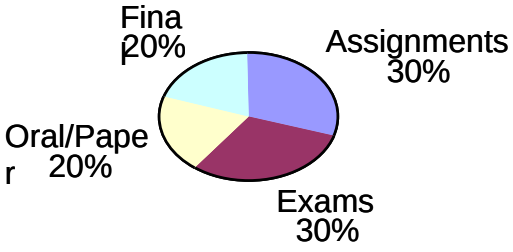
In addition, you will also write a paper summarizing your talk for distribution to the rest of the class (and me). It is due the day of your talk, and should as best as possible mirror what was discussed.

Four of these subjects are covered in the text. For the other two, you can find what you need on Braid Groups at http://math.ucr.edu/home/baez/braids/node4.html and Elliptic Curve basics are at http://www.best.com/~cgd/home/flt/flt03.htm.

Grade Breakdown

You should not expect a curve to be applied to the point scale. I know I sure don't.

Grade Distribution



Point Scale

A: 92 - ∞		A-: 90 - 91
B+: 88 - 89	B: 82 - 87	B-: 80 - 81
C+: 78 - 79	C: 72 - 77	C-: 70 - 71
D+: 68 - 69	D: 62 - 67	D-: 60 - 61
F: (-∞)- 59		

Tentative Calendar

Note: Intervals represent odd problems unless otherwise specified.

Date	Section	Problems
Aug 25	0, 1	Read these carefully
Aug 27	2	1-13, 13-21, 37
Aug 31	3	1-17, 27-31
Sep 1	4	1-19, 22, 25, 29, 31, 35
Sep 3	5	1-27, 33, 41, 47
Sep 6	6	1-29, 45, 49
Sep 8	*Q&A*	
Sep 10	8	1-13, 17-27, 41, 53
Sep 13		
Sep 15	9	1-17, 23, 29, 33
Sep 17	10	1-15, 27, 39, 41, 45
Sep 20	11	1-27, 39, 41, 47, 49, 53
Sep 22	*Q&A*	**Exam #1**
Sep 24	13	1-29, 33-47, 51
Sep 27	14	1-15, 27-31, 37
Sep 29		
Oct 1	15	1-13, 21, 25, 29, 35
Oct 4	16	1, 3, 11, 13
Oct 6		
Oct 8	*Q&A*	
Oct 11	18	1-19, 23, 27, 31, 35-43, 47

Date	Section	Problems
Oct 13	19	1-13, 23, 29
Oct 18	20	1-21, 27, 28
Oct 20	21	1, 7-11
Oct 22	22	1-17, 23-27
Oct 25	23	1-21, 27, 29, 35
Oct 27	*Q&A*	**Exam #2**
Oct 29	26	1, 3, 11, 13, 17, 19, 25, 29, 37
Nov 1	27	1-7, 15, 16, 19, 25, 29, 37
Nov 3		
Nov 5	29	1-17, 31, 35
Nov 8	30	1-9, 19-25
Nov 10	31	1-13, 19, 23, 27, 29, 37
Nov 12	32	1-5, 9
Nov 15	33	1-5, 9
Nov 17	*Q&A*	**Exam #3**
Nov 19	38	1, 3, 9, 13
Nov 22	39	1-5, 11
Nov 29	Presentation	Braid Groups, Cayley Digraphs
Dec 1	Presentation	Elliptic Curves, GA & Counting
Dec 3	Presentation	Isometries, Series of Groups
Dec 6	*Q&A*	Return papers, exams, etc.